VFO Control for Mobile Vehicles in the Presence of Skid Phenomenon

Maciej Michałek *

1 Introduction

Skid is a common phenomenon in real vehicles. Sometimes this effect is negligibly small and in a consequence can be omitted in a vehicle model preserving practically acceptable control performance. In a case of nonholonomic mobile robots it is equivalent to the *rolling without slipping* assumption. However, in many real-life situations the skid phenomenon influences vehicle motion in a such degree, that exclusion of it in a system model leads to significant control performance deterioration. The term *skid* used in this paper should be understood as a phenomenon of motion velocity disturbance, which for nonholonomic vehicles is connected with violating kinematic constraints. Wind blowing during aircraft flight, currents and waves during ship cruising, loosing adhesion between road surface and wheels during car ride or moving on sloping areas are examples of situations, where a *skid* phenomenon usually appears. To enhance tracking precision in these cases, control laws dedicated for mobile vehicles should be robust to skid-like disturbances. It is especially important when skidding is not vanishing and persistently disturbs vehicle motion.

In the literature the problem of motion-with-skid control has been considered in some papers. One can recall [1] and [6] for the case of car-like vehicles (the second one treats about an agricultural tractor), also [8] where the control task was designed for a skid-steering mobile robot, and [4] dedicated to practical issues of ship steering.

In [2] a new VFO control methodology[1] dedicated to tracking and stabilization of specific subclass of nonholonomic driftless systems has been proposed and verified. The VFO control strategy can be treated as a generalization of control in polar coordinates (the need of such a generalization has been already pointed to in [5]). The results have revealed good performance obtained for 3-dimensional systems with VFO controllers guaranteeing, among others, good control quality (fast asymptotic error convergence, natural and non-oscillatory transient behavior), intuitive control input interpretation, and very simple controller parametric synthesis. The aim of this paper is a description of the VFO tracking control strategy extension for mobile vehicles in relation to the original concept presented in [2]. The extension concerns a case of skid phenomenon influence compensation during vehicle motion. The VFO controller proposed in [2] is modified to preserve asymptotic convergence of a vehicle position error despite of skid

*Chair of Control and Systems Engineering, Poznań University of Technology (PUT), Piotrowo 3a, 60-965 Poznań, Poland, e-mail: maciej.michalek@put.poznan.pl

[1]VFO is an abbreviation from words: Vector Field(s) Orientation.

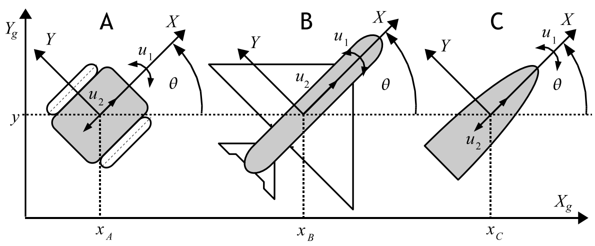

Figure 1: Three examples of mobile vehicles moving on a plane: two-wheeled robot (A), aircraft (B), ship (C)

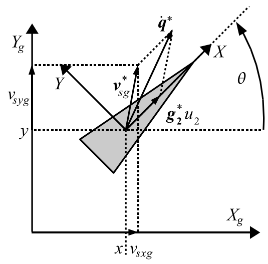

Figure 2: Mobile vehicle as a rigid body on a plane

effect existence. A similar idea connected with a drift compensation and applied to a motion planning task can be found in [3].

2 Problem formulation

Let us consider the class of restricted-mobility mobile vehicles moving on a plane, which in absence of skid can be modeled by the equation

$$\begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \Rightarrow \quad \dot{\boldsymbol{q}} = \boldsymbol{G}(\boldsymbol{q})\boldsymbol{u}, \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^3$ is a vehicle state and $\boldsymbol{u} \in \mathbb{R}^2$ is a control vector defined in a space of velocities[2]. Examples of vehicles from the considered class with geometrical interpretation of state variables and control inputs are depicted in Fig. 1.

2.1 Skid phenomenon

Since we consider a motion control problem on a kinematic level, we will not be interested in analyzing reasons of the skid phenomenon. On this level it is rather justified to take into account only direct results of skidding. Regardless of the reasons of the skid effect, the results are always the same – an additional velocity vector appears in (1):

$$\begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ v_{sxg} \\ v_{syg} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{v}_{sg}^* = [v_{sxg} \ v_{syg}]^T \in \mathbb{R}^2$ is a skid velocity vector described in a global frame (Fig. 2). Expressing the $\boldsymbol{v}_{sg}^*$ term in a local frame attached to the vehicle body one can rewrite (2) as follows[3]:

$$\begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} (u_2 + v_{sx}) + \begin{bmatrix} 0 \\ -\sin\theta \\ \cos\theta \end{bmatrix} v_{sy}, \tag{3}$$

[2]Sometimes in real vehicles one should assume $u_2 \in \mathbb{R}_+$ like in the aircraft case.

[3]Eq.(3) is equivalent to the rigid body motion $\dot{\boldsymbol{q}} = \boldsymbol{R}(\theta)\boldsymbol{\eta}$, where $\boldsymbol{R}(\theta) \in SO(3)$ and $\boldsymbol{\eta} = [u_1 \ (u_2 + v_{sx}) \ v_{sy}]^T \in \mathbb{R}^3$.

where $[v_{sx}\ v_{sy}]^T = \boldsymbol{v}_s^* \in \mathbb{R}^2$ is the skid velocity vector described in a local frame (one can treat Eq.(3) as a two-input control system with a drift term $\boldsymbol{g}_0 = [0\ -v_{sy}\sin\theta\ v_{sy}\cos\theta]^T$). From Eq.(3) it follows that the skid effect can be considered as a violation of a nonholonomic constraint connected with system (1) – due to the nonzero lateral velocity component v_{sy} – and also as a longitudinal velocity disturbance – due to the nonzero v_{sx} term. Depending on the reasons causing the skid, terms v_{sx} and v_{sy} can be treated as constant or varying in time during vehicle movement[4]. For example, directional wind or current disturbing a ship cruise (see [4]) or a long-time motion along a slope give constant skid influence in a global frame, so in a local frame skid velocity coordinates can be varying. The opposite situation can be seen for instance during mo- ving along a curve on a slippery ground, where one can notice constant lateral deviation of the vehicle velocity [6]. It is worth mentioning that partially also dynamic effects not included in kinematics (1) but appearing in experiments can be treated as a *virtual skid* included in the v_{sx} element. Now, let us formulate a control problem, which is intended to be solved in the sequel.

Problem 1 *For a given admissible and persistently exciting reference trajectory* $\boldsymbol{q}_t(\tau) = [\theta_t(\tau)\ x_t(\tau)\ y_t(\tau)]^T \in \mathbb{R}^3$ *and assuming a nonzero skid velocity* $\boldsymbol{v}_{sg}^* \in \mathcal{L}_\infty$ *with* $\dot{\boldsymbol{v}}_{sg}^* \in \mathcal{L}_\infty$*, find a bounded control law* $\boldsymbol{u}(\boldsymbol{q}_t, \boldsymbol{q}, \cdot)$ *for the kinematics (2), which guarantees asymptotic convergence of the position error* $\boldsymbol{e}^* = [e_x\ e_y]^T \in \mathbb{R}^2$ *to zero and boundedness of the orientation error* e_θ *in the sense that* $\lim_{\tau\to\infty} \boldsymbol{e}^*(\tau) = \boldsymbol{0}$*,* $e_\theta(\tau) \in \mathcal{L}_\infty$*, where:*

$$e_x \triangleq x_t - x, \qquad e_y \triangleq y_t - y, \qquad e_\theta \triangleq \theta_t - \theta. \tag{4}$$

The admissibility of a reference trajectory means that $\boldsymbol{q}_t(\tau)$ satisfies Eq.(1) for some reference inputs $u_{1t}(\tau), u_{2t}(\tau) \in \mathcal{L}_\infty$. Persistent excitation implies that $\forall_{\tau\geqslant 0}\, u_{2t}(\tau) \neq 0$ (see [2]).

3 VFO controller

First of all we recall the basic idea of the VFO control strategy dedicated to kinematics (1), hence for a case without skid disturbances. After that, by analogy to previous considerations, an extension regarding a nonzero skid velocity will be presented.

3.1 VFO strategy – brief recall

It has been shown (for example in [2]) that the VFO control strategy results from simple geometrical interpretations and can be applied to a subclass of nonholonomic driftless systems with two inputs[5]. System (1) belongs to this class. One can decompose (1) into two subsystems: one-dimensional *orienting subsystem* represented by the equation $\dot{\theta} = u_1$ and two-dimensional *pushing subsystem* represented by the equation $\dot{\boldsymbol{q}}^* = \boldsymbol{g}_2^* u_2$, where $\boldsymbol{q}^* = [x\ y]^T$ and $\boldsymbol{g}_2^* = [\cos\theta\ \ \sin\theta]^T$. Since the direction (and orientation) of vector $\boldsymbol{g}_2^*(\theta)$ (and in a consequence – of velocity $\dot{\boldsymbol{q}}^*(\theta)$) depends on the θ variable, it has been proposed to call θ the *orienting variable*, and the u_1 input – the *orienting*

[4]In real conditions also random fluctuations of skid components are characteristic.

[5]The VFO control can be treated as a generalization of control in polar coordinates.

control. Follo- wing this interpretation the u_2 input has been called the *pushing control*, since it pushes the sub-state $\boldsymbol{q}^*$ along the current direction of $\boldsymbol{g}_2^*$. The proposed control law for a tracking task has been derived from the desired relation between the $\boldsymbol{g}_2$ vector field and introduced additional *convergence vector field* $\boldsymbol{h} = [h_1 \ \boldsymbol{h}^{*T}]^T$. This vector field has been designed so as to indicate in any state $\boldsymbol{q}$ an instantaneous convergence direction to a reference trajectory $\boldsymbol{q}_t$. In this way the VFO control strategy can be decomposed into the *orienting subprocess*, responsible for putting the direction of $\boldsymbol{g}_2^*$ onto the $\boldsymbol{h}^*$ vector (with the u_1 input), and the *pushing subprocess* responsible for pushing the state $\boldsymbol{q}^*$ (with the u_2 input) to the reference trajectory $\boldsymbol{q}_t^*$. Since the orienting variable θ plays an auxiliary role during the orienting subprocess, only a proper construction of the $\boldsymbol{h}$ vector will guarantee asymptotic convergence also of the θ variable to the reference signal θ_t near the reference trajectory $\boldsymbol{q}_t^*$ (for details the reader is referred to [2]).

3.2 VFO control with skid compensation

Considering the skid phenomenon, we have to use model (2) or (3) instead of kinematics (1). By analogy to the comments of Section 3.1 we introduce the convergence vector field $\boldsymbol{h} = [h_1 \ \boldsymbol{h}^{*T}]^T \in \mathbb{R}^3$, where

$$\boldsymbol{h}^*(\boldsymbol{e}^*, \dot{\boldsymbol{q}}_t^*) = \begin{bmatrix} h_2 & h_3 \end{bmatrix}^T \triangleq k_p \, \boldsymbol{e}^* + \dot{\boldsymbol{q}}_t^*, \qquad \dot{\boldsymbol{q}}_t^* = \begin{bmatrix} \dot{x}_t & \dot{y}_t \end{bmatrix}^T, \tag{5}$$

and $k_p > 0$ is a design parameter (definition of the h_1 component will be introduced later). Since for every error $\boldsymbol{e}^*$, vector $\boldsymbol{h}^*(\boldsymbol{e}^*, \dot{\boldsymbol{q}}_t^*)$ defines a desired direction (and orientation) of motion for system (2), the VFO strategy demands meeting the following convergence relation:

$$\dot{\boldsymbol{q}}^*(\tau) \overset{\tau \to \infty}{\longrightarrow} f_k(\tau) \, \boldsymbol{h}^*(\boldsymbol{e}^*(\tau), \dot{\boldsymbol{q}}_t^*(\tau)), \tag{6}$$

where $f_k(\tau)$ is a some scalar and nonzero smooth function. Relation (6) describes the desirable situation, where the instantaneous directions of vectors $\dot{\boldsymbol{q}}^*$ and $\boldsymbol{h}^*$ are matched (subsystem $\dot{\boldsymbol{q}}^* = \boldsymbol{g}_2^* u_2 + \boldsymbol{v}_{sg}^*$ evolves along the convergence direction defined by $\boldsymbol{h}^*$). From now on we take $f_k(\tau) = f_k \equiv 1$ to stress that not only directions but also orientations and norms of $\dot{\boldsymbol{q}}^*$ and $\boldsymbol{h}^*$ should be matched. Substituting the particular terms from (2) and (5) into (6) gives

$$\lim_{\tau \to \infty} \left\{ \begin{array}{rcl} u_2(\tau) \cos\theta(\tau) + v_{sxg}(\tau) - h_2(\tau) & = & 0 \\ u_2(\tau) \sin\theta(\tau) + v_{syg}(\tau) - h_3(\tau) & = & 0 \end{array} \right. ,$$

which can be rewritten in a more compact form as

$$\lim_{\tau \to \infty} \left[\mathrm{Atan2}\left(H_3 \,\mathrm{sgn}(u_2), H_2 \,\mathrm{sgn}(u_2) \right) - \theta(\tau) \right] = 0 \tag{7}$$

with

$$H_2 \triangleq h_2 - v_{sxg}, \qquad H_3 \triangleq h_3 - v_{syg}. \tag{8}$$

The limit (7) describes the so-called *orienting condition*, which gua- rantees matching of orientations for vectors $\dot{\boldsymbol{q}}^*$ and $\boldsymbol{h}^*$. Hereafter the VFO control design follows by analogy to the original concept mentio- ned in Section 3.1. Recalling (7), we introduce an auxiliary variable

$$\theta_a \triangleq \mathrm{Atan2c}\left(H_3 \,\mathrm{sgn}(u_{2t}), H_2 \,\mathrm{sgn}(u_{2t}) \right) \in \mathbb{R}, \tag{9}$$

where Atan2c $(\cdot,\cdot) : \mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is a continuous version of the function Atan2 $(\cdot,\cdot) : \mathbb{R}\times\mathbb{R}\to(-\pi,\pi]$, and the term $\text{sgn}(u_2)$ was replaced with $\text{sgn}(u_{2t})$ to guarantee proper transient behavior of a closed-loop system[6]. To meet the relation (7) it suffices to make the auxiliary error $e_a = \theta_a - \theta$ tend to zero. Hence, let us define the h_1 component of vector $\boldsymbol{h}$ as follows:

$$h_1 \triangleq k_1 e_a + \dot{\theta}_a, \tag{10}$$

where $k_1 > 0$ is a second design coefficient, $\dot{\theta}_a = (\dot{H}_3 H_2 - H_3\dot{H}_2)/\|\boldsymbol{H}^*\|^2$ results from time-differentiation of (9), and $\boldsymbol{H}^* = [H_2\ H_3]^T$. According to the VFO strategy one proposes the following VFO control law:

$$u_1 = h_1 \quad\Rightarrow\quad u_1 = k_1 e_a + \dot{\theta}_a, \tag{11}$$

$$u_2 = \boldsymbol{g}_2^{*T}\boldsymbol{H}^* \quad\Rightarrow\quad u_2 = H_2\cos\theta + H_3\sin\theta. \tag{12}$$

Equation (12) comes from an idea of *careful pushing*, which allows for the fastest pushing only when the orienting condition (7) – and equiva- lently relation (6) – is satisfied.

Proposition 1 *Assuming that* $\forall_{\tau\geqslant 0}\ \boldsymbol{H}^*(\tau)\neq\boldsymbol{0}$ *and the reference trajectory* $\boldsymbol{q}_t^*$ *is sufficiently smooth:* $\dot{\boldsymbol{q}}_t^*, \ddot{\boldsymbol{q}}_t^* \in \mathcal{L}_\infty$*, the VFO control law (11,12) applied to system (2) solves Problem 1.*

Proof. Let us first consider behavior of the auxiliary error $e_a = \theta_a - \theta$. Substituting (11) into (2) yields the equation $\dot{e}_a + k_1 e_a = 0$, which shows exponential convergence of the θ variable to the auxiliary one θ_a. As a second stage we consider evolution of the $\boldsymbol{e}^* = [e_x\ e_y]^T$ error. From (4), (5), and (8) one can write

$$\boldsymbol{H}^* = k_p\boldsymbol{e}^* + \dot{\boldsymbol{q}}_t^* - \boldsymbol{v}_{sg}^*, \qquad \dot{\boldsymbol{e}}^* = -k_p\boldsymbol{e}^* + \boldsymbol{r}, \qquad \boldsymbol{r} = \boldsymbol{H}^* - \boldsymbol{g}_2^* u_2. \tag{13}$$

It can be shown that the following expressions are true:

$$\|\boldsymbol{r}\| = \|\boldsymbol{H}^*\|\,\gamma(\theta) \qquad \text{and} \qquad \lim_{\theta\to\theta_a}\gamma(\theta) = 0, \tag{14}$$

where $\gamma(\theta) = \sqrt{1-\cos^2\alpha(\theta)} \in [0,1]$ and $\alpha(\theta) = \angle(\boldsymbol{g}_2^*(\theta), \boldsymbol{H}^*)$. Introducing a positive definite function $V = \frac{1}{2}(\boldsymbol{e}^{*T}\boldsymbol{e}^*)$ and using (13,14) one can assess its time derivative as follows:

$$\begin{aligned}
\dot{V} &= \boldsymbol{e}^{*T}\dot{\boldsymbol{e}}^* = \boldsymbol{e}^{*T}\left(-k_p\boldsymbol{e}^* + \boldsymbol{r}\right) = -k_p\|\boldsymbol{e}^*\|^2 + \boldsymbol{e}^{*T}\boldsymbol{r} \leqslant \\
&\leqslant -k_p\|\boldsymbol{e}^*\|^2 + \|\boldsymbol{e}^*\|\,\|\boldsymbol{r}\| = -k_p\|\boldsymbol{e}^*\|^2 + \|\boldsymbol{e}^*\|\,\|\boldsymbol{H}^*\|\,\gamma = \\
&= -k_p\|\boldsymbol{e}^*\|^2 + \|\boldsymbol{e}^*\|\,\left\|k_p\boldsymbol{e}^* + \dot{\boldsymbol{q}}_t^* - \boldsymbol{v}_{sg}^*\right\|\gamma \leqslant \\
&\leqslant -k_p(1-\gamma)\|\boldsymbol{e}^*\|^2 + \gamma\|\boldsymbol{e}^*\|\,\kappa = -W(\boldsymbol{e}^*, \dot{\boldsymbol{q}}_t^*, \boldsymbol{v}_{sg}^*, \gamma),
\end{aligned}$$

where $\kappa = \|\dot{\boldsymbol{q}}_t^*\| + \left\|\boldsymbol{v}_{sg}^*\right\|$. Function $W(\tau)$ is positive definite for

$$\|\boldsymbol{e}^*(\tau)\| > \Gamma(\tau), \quad \text{where} \quad \Gamma(\tau) = \frac{\gamma(\tau)\kappa(\tau)}{k_p(1-\gamma(\tau))}. \tag{15}$$

[6]The term $\text{sgn}(u_{2t})$ remains constant during a tracking task for the considered persi- stently exciting reference trajectories (see the comment after Problem 1).

Function Γ is finite for $\gamma < 1$. The case $\gamma = 1$ in (15) is possible, but is temporary and non-attractive. It can occur only during a transient stage (for some $\tau = \overline{\tau} < \infty$). Moreover, $W(\overline{\tau}) = -\|\boldsymbol{e}^*(\overline{\tau})\|\,\kappa(\overline{\tau}) < \infty \;\Rightarrow\; \dot{V}(\overline{\tau}) < \infty$ (finite time escape for $\|\boldsymbol{e}^*\|$ when $\gamma = 1$ is not possible). Since $\gamma(\tau)$ can never get stuck in $\gamma = 1$ and $\exists_{\tau_\gamma < \infty} : \forall_{\tau > \tau_\gamma} \gamma(\tau) < 1$ (as a direct consequence of (14) and exponential convergence of e_a) and since $\kappa \in \mathcal{L}_\infty$ (from assumption), one can conclude that (15) is determined for finite $\Gamma(\tau)$ almost always and $\forall\, \tau > \tau_\gamma$. As a consequence $\|\boldsymbol{e}^*(\tau)\| \in \mathcal{L}_\infty$ and also $V, W, \|\boldsymbol{H}^*\|, \|\boldsymbol{r}\| \in \mathcal{L}_\infty$. Now, from (13) $\|\dot{\boldsymbol{e}}^*\| \in \mathcal{L}_\infty$ and hence $\dot{V} \in \mathcal{L}_\infty$. Since it can be shown that $W(\tau)$ is uniformly continuous and integrible for $\tau \in [0, \infty)$, the Barbalat's lemma implies $\lim_{\tau\to\infty} W(\tau) = 0 \;\Rightarrow\; \lim_{\tau\to\infty}[\|\boldsymbol{e}^*(\tau)\| - \Gamma(\tau)] = 0$. Recalling the limit from (14) and since $\lim_{\tau\to\infty} e_a(\tau) = 0$, one concludes: $\lim_{\tau\to\infty} \gamma(\tau) = 0 \;\Rightarrow\; \lim_{\tau\to\infty} \Gamma(\tau) = 0$ and finally $\lim_{\tau\to\infty} \|\boldsymbol{e}^*(\tau)\| = 0$. The boundedness of θ_a and θ_t together with the exponential convergence of e_a implies $\theta \in \mathcal{L}_\infty$, and $e_\theta \in \mathcal{L}_\infty$. $\square$

Remark 1. Control function (11) has a discontinuous nature. It results from definition (9), which is not determined for $\boldsymbol{H}^* = \boldsymbol{0}$ (the reason of the assum- ption in Proposition 1). The equality $\boldsymbol{H}^* = \boldsymbol{0}$ relates to two cases (compare (13) and (5)): A) for $\boldsymbol{e}^* \neq \boldsymbol{0} \wedge \boldsymbol{v}^*_{sg} = \boldsymbol{h}^*$ – only during a transient stage or B) for $\boldsymbol{e}^* = \boldsymbol{0} \wedge \boldsymbol{v}^*_{sg} = \dot{\boldsymbol{q}}^*_t$ – when a system evolves exactly along a reference trajectory $\boldsymbol{q}^*_t$. Since for $\boldsymbol{H}^* = \boldsymbol{0}$ also $u_2 = 0$ (see (12)), the only term which drives a subsystem $\dot{\boldsymbol{q}}^* = \boldsymbol{g}^*_2 u_2 + \boldsymbol{v}^*_{sg}$ is the skid velocity $\boldsymbol{v}^*_{sg}$. Hence, both cases describe a situation when the skid velocity $\boldsymbol{v}^*_{sg}$ drives a system toward[7] $\boldsymbol{q}^*_t$ (case A) or along $\boldsymbol{q}^*_t$ (case B). Moreover, for case A it can be seen from (13) that for $\boldsymbol{H}^* = \boldsymbol{0}$ holds: $\boldsymbol{r} = \boldsymbol{0} \;\Rightarrow\; \dot{\boldsymbol{e}}^* + k_p \boldsymbol{e}^* = \boldsymbol{0}$. As a consequence, the boundedness and convergence of $\boldsymbol{e}^*$ are still preserved. Both cases are connected with a u_1 control discontinuity set, but they seem to be non-attractive, non-persistent, and unlikely in practice. However, to obtain a well-defined control u_1, one proposes to introduce additional definitions for θ_a and $\dot{\theta}_a$ in assumed sufficiently small ϵ-vicinity of $\boldsymbol{H}^* = \boldsymbol{0}$. One proposes to take

$$\theta_a \triangleq \theta_a(\tau_-) \quad \text{and} \quad \dot{\theta}_a \triangleq 0 \qquad \text{for} \quad \|\boldsymbol{H}^*\| \leqslant \epsilon, \tag{16}$$

where $0 < \epsilon < \inf_\tau \left\| \dot{\boldsymbol{q}}^*_t(\tau) - \boldsymbol{v}^*_{sg}(\tau) \right\|$ and τ_- denotes the time instant of reaching the ϵ-vicinity. These additional definitions together with (9) allow the control function (11) to remain unchanged.

Remark 2. Since the H_2, H_3 and $\dot{H}_2, \dot{H}_3$ terms are used in the control law (the later two in the $\dot{\theta}_a$ term), one has to be able to compute or measure skid components v_{sxg}, v_{syg} and their time derivatives. It is a serious practical challenge to obtain these quantities. Estimating time derivatives of skid components seems rather impractical in a noisy environment. Hence, assuming relatively slow variation of skid components in comparison to other derivatives in the $\dot{\theta}_a$ term, it is justified in practice to take these quantities as equal to zero. The problem of skid computation will be considered in the next section.

3.3 Skid computation

We assume that the state variables θ, x, y of a vehicle can be measured with a sampling interval T_p using for example some exteroceptive sensor (e.g. a vision system). With this assumption the simplest way to obtain skid components v_{sxg}, v_{syg} is to estimate them

[7]Note that $\boldsymbol{h}^*$ defines an instantaneous convergence direction.

from Eq.(2). In practice, the estimates computed in such a way result from the following equations:

$$\hat{v}_{sxg}(n) = \dot{x}(n) - u_2(n-1)\cos\left[\theta(n-1) + \varepsilon_\theta(n-1)\right] + \varepsilon_x(n), \qquad (17)$$

$$\hat{v}_{syg}(n) = \dot{y}(n) - u_2(n-1)\sin\left[\theta(n-1) + \varepsilon_\theta(n-1)\right] + \varepsilon_y(n), \qquad (18)$$

where $\dot{x}(n) = [x(n) - x(n-1)]/T_p$, $\dot{y}(n) = [y(n) - y(n-1)]/T_p$, $u_2(n)$ is a pushing input sample from (12) and $\varepsilon_\theta, \varepsilon_x, \varepsilon_y$ denote random measurement noise always present in practical implementations. If noise causes high estimate variances, one can filter right-hand sides of (17,18) and use filtered components $\hat{v}^F_{sxg}, \hat{v}^F_{syg}$ (filtered-estimator). The skid estimates can now be used in definitions (8) instead of *true* signals (v_{sxg}, v_{syg}), which are not known in practice.

4 Simulation results

The verification of the effectiveness of the proposed controller has been carried out for the kinematics of a differentially driven two-wheeled mobile vehicle. To make simulation results more realistic, two main practical issues have been taken into account: 1) a limitation on the maximum vehicle wheel velocity ω_{wmax} has been imposed[8] with $\omega_{wmax} = 81$[rad/s], 2) the skid filtered-estimator[9] (17,18) computed with a sample time $T_p = 0.01$[s] has been used with measurement zero-mean Gaussian errors $\varepsilon_x, \varepsilon_y$, and ε_θ with variances $\sigma^2 = 0.01$ included in Eqs.(17,18). For simulation tests a circle-like trajectory computed for $u_{1t} = 0.5$[rad/s], $u_{2t} = 1.2$[m/s], and $\boldsymbol{q}_t(0) = [\pi/4 \; 0 \; 0]^T$ has been chosen as a reference trajectory. Moreover, the following values have been chosen: $k_p = 5$, $k_1 = 10$, $\boldsymbol{q}(0) = [0 \; -2 \; 1]^T$. During simulation the constant skid with components $v_{sxg} = v_{syg} = 0.5$[m/s] has appeared in the time instant $t_1 = 5$[s]. The results obtained[10] are illustrated in Fig. 3. It can be seen that position tracking errors tend toward zero quite fast and remain there also after the skid appearance (starting from $t_1 = 5$[s]). It is worth to note the compensative action of control signals and bounded but compensative behavior of the vehicle orientation (*crabwise* motion). The bottom-left plot shows the vehicle path on a plane, which can be compared with the bottom-right plot obtained for a case without skid compensation ($v_{sxg} = v_{syg} \equiv 0$ in (8)).

5 Concluding remarks

The VFO position tracking control with skid effect compensation for kinematic mobile vehicles has been presented. The proposed control strategy results from an extension of the original concept described in [2]. The extension can be treated as a first attempt to use the VFO control method for dynamical systems with a drift. The simulation results included in the paper illustrate the effectiveness of the proposed VFO control scheme and also reveal its relative robustness to measurement noises and control input limitations.

[8] The control vector scaling procedure described in [7] has been used.

[9] A first order low-pass filter with a time constant $T = 0.1$ has been applied.

[10] Simulations have been performed with Matlab/Simulink software.

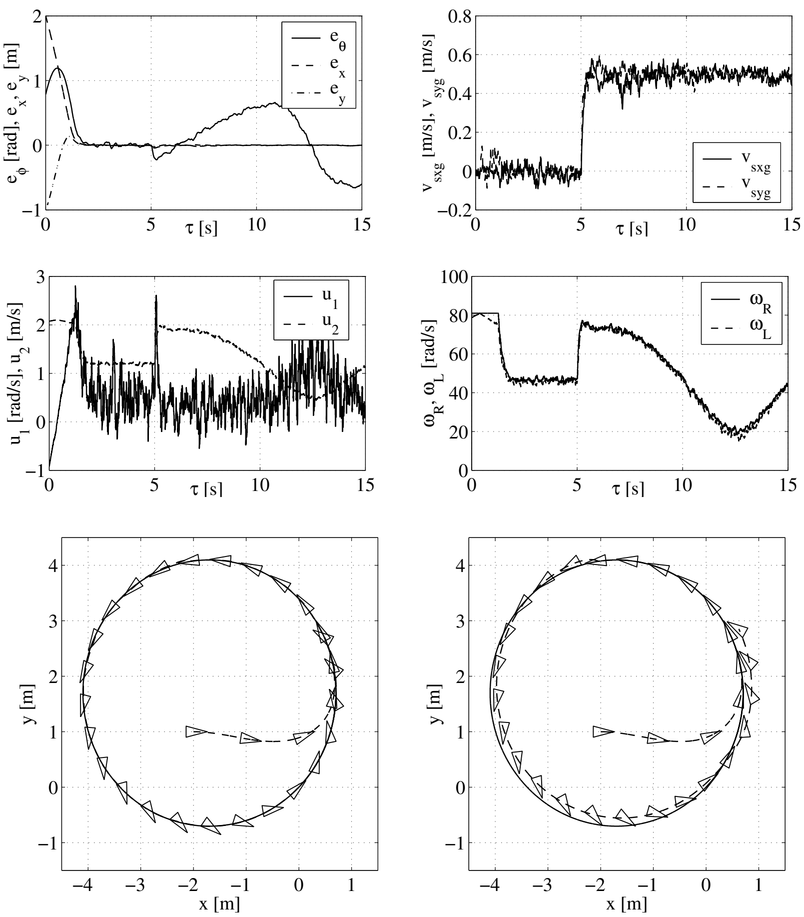

Figure 3: Tracking error, skid estimate, and control signal time-plots – top four plots; vehicle (- -) and reference (-) paths on a plane – bottom two plots (the last one on the right-hand side for the case without skid compensation)

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