# Corrigendum to "Robust output-feedback cascaded tracking controller for spatial motion of anisotropically-actuated vehicles" [Aerospace Science and Technology 92 (2019), 915-929]

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In the original paper [1], a part of the stability analysis presented in Section 4.2 is not fully correct. We provide here a corrigendum to this part which, however, *does not change the main result of work* [1] stated in Proposition 1, and *does not affect validity of numerical and experimental results* presented in [1]. For clarity, we will refer to numbers of equations introduced in [1] by using the brackets $\langle\cdot\rangle$. The paragraph between Eqs. $\langle 66\rangle$ and $\langle 74\rangle$ in [1] should be replaced by the following corrected analysis.

Assume that the state and the perturbing term of $\Sigma_1$ belong to some predefined admissible domains, i.e.,

$$\tilde{\boldsymbol{\chi}} \in \mathcal{D}_X, \ \dot{\boldsymbol{d}} \in \mathcal{D}_D,$$

where $\mathcal{D}_X = \{\tilde{\boldsymbol{\chi}} \in \mathbb{R}^{18} : \ \|\tilde{\boldsymbol{\chi}}\| < r_X\}$, $\mathcal{D}_D = \{\dot{\boldsymbol{d}} \in \mathbb{R}^6 : \ \|\dot{\boldsymbol{d}}\| < r_D\}$ for some positive constants $r_X$ and $r_D$. Since the matrix $\boldsymbol{H}$ introduced in $\langle 61\rangle$ is Hurwitz but it is neither symmetric nor positive definite, we cannot make an upper bound estimation as in $\langle 68\rangle$. Therefore, we propose to perform a stability analysis using auxiliary dynamics. To this aim, let us introduce an auxiliary transformation of variables

$$\tilde{\boldsymbol{\chi}} := \boldsymbol{L}_\zeta \boldsymbol{\zeta} : \ \mathcal{D}_\zeta \to \mathcal{D}_X, \tag{1}$$

where $\mathcal{D}_\zeta \triangleq \{\boldsymbol{\zeta} \in \mathbb{R}^{18} : \|\boldsymbol{\zeta}\| < \|\boldsymbol{L}_\zeta^{-1}\| r_X =: r_Z\}$, and $\boldsymbol{L}_\zeta \triangleq \mathrm{blkdiag}\{\boldsymbol{W}_\omega^{-2}, \boldsymbol{W}_\omega^{-1}, \boldsymbol{I}\}$, $\boldsymbol{W}_\omega \triangleq \mathrm{diag}\{\omega_{o1}, \ldots, \omega_{o6}\} \succ 0$, thus $\|\boldsymbol{L}_\zeta^{-1}\| = \max\{1, \Omega_o^2\}$, $\Omega_o \triangleq \max\{\omega_{o1}, \ldots, \omega_{o6}\}$. By referring to dynamics $\langle 61\rangle$ and the synthesis rule $\langle 58\rangle$, one can express the auxiliary dynamics $\dot{\boldsymbol{\zeta}} = -\boldsymbol{L}_\zeta^{-1}\boldsymbol{H}\boldsymbol{L}_\zeta\boldsymbol{\zeta} + \boldsymbol{L}_\zeta^{-1}\boldsymbol{I}_\chi\dot{\boldsymbol{d}}$ in the form

$$\dot{\boldsymbol{\zeta}} = -\boldsymbol{L}_\omega \boldsymbol{H}_\zeta \boldsymbol{\zeta} + \boldsymbol{I}_\chi \dot{\boldsymbol{d}}, \tag{2}$$

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where $\boldsymbol{I}_\chi = [\mathbf{0}\ \mathbf{0}\ \boldsymbol{I}]^\top$ is the *input matrix* introduced in $\langle 61 \rangle$, while $\boldsymbol{L}_\omega = \mathrm{blkdiag}\{\boldsymbol{W}_\omega, \boldsymbol{W}_\omega, \boldsymbol{W}_\omega\}$ and

$$\boldsymbol{H}_\zeta = \begin{bmatrix} 3\boldsymbol{I} & -\boldsymbol{I} & \mathbf{0} \\ 3\boldsymbol{I} & \mathbf{0} & -\boldsymbol{I} \\ \boldsymbol{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

Let us define a positive definite function $V_\zeta \triangleq \boldsymbol{\zeta}^\top \boldsymbol{P}_\zeta \boldsymbol{\zeta}$ which satisfies $\alpha_{1\zeta}(\|\boldsymbol{\zeta}\|) \leq V_\zeta \leq \alpha_{2\zeta}(\|\boldsymbol{\zeta}\|)$, where $\alpha_{1\zeta}(\|\boldsymbol{\zeta}\|) = \lambda_{\min}(\boldsymbol{P}_\zeta)\|\boldsymbol{\zeta}\|^2$ and $\alpha_{2\zeta}(\|\boldsymbol{\zeta}\|) = \lambda_{\max}(\boldsymbol{P}_\zeta)\|\boldsymbol{\zeta}\|^2$, while $\boldsymbol{P}_\zeta = \boldsymbol{P}_\zeta^\top \succ 0$ is a solution (independent of $\omega_{oi}$) of the Lyapunov equation $\boldsymbol{H}_\zeta^\top \boldsymbol{L}_\omega^\top \boldsymbol{P}_\zeta + \boldsymbol{P}_\zeta \boldsymbol{L}_\omega \boldsymbol{H}_\zeta = \boldsymbol{L}_\omega$. By using dynamics (2), one can estimate a time derivative of $V_\zeta$ as follows: $\dot{V}_\zeta \leq -(1-\nu_\zeta)\omega_o \|\boldsymbol{\zeta}\|^2 + \|\boldsymbol{\zeta}\|(2\lambda_{\max}(\boldsymbol{P}_\zeta)\|\dot{\boldsymbol{d}}\| - \nu_\zeta \omega_o \|\boldsymbol{\zeta}\|)$, where $\omega_o \triangleq \min\{\omega_{o1}, \ldots, \omega_{o6}\}$ and $\nu_\zeta \in (0,1)$ is a majorization constant. Thus,

$$\dot{V}_\zeta \leq -(1-\nu_\zeta)\omega_o \|\boldsymbol{\zeta}\|^2 \quad \text{if} \quad \|\boldsymbol{\zeta}\| \geq \frac{2\lambda_{\max}(\boldsymbol{P}_\zeta)\|\dot{\boldsymbol{d}}\|}{\nu_\zeta \omega_o} =: \chi_\zeta(\|\dot{\boldsymbol{d}}\|),$$

and one concludes that auxiliary dynamics (2) is locally ISS with respect to the perturbing input $\dot{\boldsymbol{d}}$, that is, for all $t \geq 0$

$$\|\boldsymbol{\zeta}(t)\| \leq \beta_\zeta(\|\boldsymbol{\zeta}(0)\|, t) + \gamma_\zeta\left(\sup_{t\geq 0}\|\dot{\boldsymbol{d}}(t)\|\right),$$

for some $\mathcal{KL}$-class function $\beta_\zeta(\cdot,\cdot)$ and function

$$\gamma_\zeta(\|\dot{\boldsymbol{d}}\|) = \alpha_{1\zeta}^{-1}(\alpha_{2\zeta}(\chi_\zeta(\|\dot{\boldsymbol{d}}\|))) = \sqrt{\frac{\lambda_{\max}(\boldsymbol{P}_\zeta)}{\lambda_{\min}(\boldsymbol{P}_\zeta)}}\chi_\zeta(\|\dot{\boldsymbol{d}}\|),$$

if

$$\|\boldsymbol{\zeta}(0)\| < \alpha_{2\zeta}^{-1}(\alpha_{1\zeta}(r_Z)) = \sqrt{\frac{\lambda_{\min}(\boldsymbol{P}_\zeta)}{\lambda_{\max}(\boldsymbol{P}_\zeta)}} r_Z =: r_\zeta, \tag{3}$$

$$\sup_{t\geq 0}\|\dot{\boldsymbol{d}}(t)\| < \chi_\zeta^{-1}(\min\{r_\zeta, \chi_\zeta(r_D)\}) =: r_d, \tag{4}$$

where $r_Z = r_X \max\{1, \Omega_o^2\}$. According to the asymptotic gain property of ISS dynamics, one can write (using a short notation $\mathrm{ls}_\infty \equiv \limsup_{t\to\infty}$)

$$\mathrm{ls}_\infty \|\boldsymbol{\zeta}(t)\| \leq \gamma_\zeta(\mathrm{ls}_\infty \|\dot{\boldsymbol{d}}(t)\|) < \frac{2\lambda_{\max}^{3/2}(\boldsymbol{P}_\zeta)}{\lambda_{\min}^{1/2}(\boldsymbol{P}_\zeta)} \cdot \frac{r_d}{\nu_\zeta \omega_o}. \tag{5}$$

Now, by recalling (1) and observing that $\|\boldsymbol{L}_\zeta\| = \max\{1, \omega_o^{-2}\}$, one concludes upon (5) what follows:

$$\mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\| \leq \|\boldsymbol{L}_\zeta\| \cdot \mathrm{ls}_\infty \|\boldsymbol{\zeta}(t)\| < \frac{\varsigma r_d}{\nu_\zeta \omega_o}, \tag{6}$$

where

$$\varsigma = \max\{1, \omega_o^{-2}\} \frac{2\lambda_{\max}^{3/2}(\boldsymbol{P}_\zeta)}{\lambda_{\min}^{1/2}(\boldsymbol{P}_\zeta)}$$

(which is constant and independent of $\omega_o$ for $\omega_o \geq 1$), if (upon (1) and (3))

$$\|\tilde{\boldsymbol{\chi}}(0)\| < \sqrt{\frac{\lambda_{\min}(\boldsymbol{P}_\zeta)}{\lambda_{\max}(\boldsymbol{P}_\zeta)}} \frac{r_Z}{\|\boldsymbol{L}_\zeta^{-1}\|} = \sqrt{\frac{\lambda_{\min}(\boldsymbol{P}_\zeta)}{\lambda_{\max}(\boldsymbol{P}_\zeta)}} r_X. \tag{7}$$

[Please, compare the corrected results (6), (4) and (7), respectively, with ⟨74⟩, ⟨73⟩, and ⟨72⟩ presented in [1]. One can observe that by increasing $\omega_o$ the main corrected result (6) differs from ⟨74⟩ only by some resultant scaling factor.]

As a consequence, the corrected result (6) affects the forms of estimated terminal upper bounds ⟨78⟩, ⟨82⟩, and ⟨87⟩ presented in [1]. Their corrected forms are the following:

$$\begin{aligned}\mathrm{ls}_\infty \|\boldsymbol{\epsilon}(t)\| &\leq \gamma_\epsilon(\mathrm{ls}_\infty \|\boldsymbol{\delta}_\epsilon(t)\|) \leq \gamma_\epsilon(\mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|) \\ &= \frac{1+\|\boldsymbol{K}\|}{\nu_\epsilon \lambda_{\min}(\boldsymbol{K})} \mathrm{ls}_\infty \|\boldsymbol{\chi}(t)\| < \frac{1+\|\boldsymbol{K}\|}{\nu_\epsilon \lambda_{\min}(\boldsymbol{K})} \frac{\varsigma r_d}{\nu_\zeta \omega_o},\end{aligned}$$

$$\begin{aligned}\mathrm{ls}_\infty \|\bar{\boldsymbol{e}}_a(t)\| &\leq \gamma_a(\mathrm{ls}_\infty \|\boldsymbol{\delta}_a(t)\|) \leq \gamma_a(\mathrm{ls}_\infty \|\boldsymbol{\epsilon}(t)\|) + \gamma_a(\mathrm{ls}_\infty \|\dot{\tilde{\boldsymbol{\eta}}}_{oa}(t)\|) \\ &\leq \gamma_a(\gamma_\epsilon(\mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|)) + \gamma_a(k_p \Theta_a \mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|) \\ &= \frac{2}{\nu_a k_a}\left(\frac{1+\|\boldsymbol{K}\|}{\nu_\epsilon \lambda_{\min}(\boldsymbol{K})} + k_p \Theta_a\right) \mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\| \\ &< \frac{2}{\nu_a k_a}\left(\frac{1+\|\boldsymbol{K}\|}{\nu_\epsilon \lambda_{\min}(\boldsymbol{K})} + k_p \Theta_a\right) \frac{\varsigma r_d}{\nu_\zeta \omega_o} =: r_{ea}^\infty,\end{aligned}$$

and

$$\begin{aligned}\mathrm{ls}_\infty \|\boldsymbol{e}_p(t)\| &\leq \gamma_p(\mathrm{ls}_\infty \|\boldsymbol{\delta}_p(t)\|) \\ &\leq \gamma_p\left[\gamma_a(\mathrm{ls}_\infty \|\boldsymbol{\epsilon}(t)\|) + \gamma_a(\mathrm{ls}_\infty \|\dot{\tilde{\boldsymbol{\eta}}}_{oa}(t)\|) + \mathrm{ls}_\infty \|\boldsymbol{\epsilon}(t)\|\right] \\ &\leq \gamma_p\left[\gamma_a(\gamma_\epsilon(\mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|)) + \gamma_a(k_p \Theta_a \mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|) + \gamma_\epsilon(\mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|)\right] \\ &= \gamma_p\left[\left(\frac{2(1+\|\boldsymbol{K}\|)}{k_a \nu_a \nu_\epsilon \lambda_{\min}(\boldsymbol{K})} + \frac{2k_p \Theta_a}{k_a \nu_a} + \frac{1+\|\boldsymbol{K}\|}{\nu_\epsilon \lambda_{\min}(\boldsymbol{K})}\right) \mathrm{ls}_\infty \|\tilde{\boldsymbol{\chi}}(t)\|\right] \\ &< \frac{\varsigma r_d(\rho_1+\rho_2+\rho_3)(1+\sqrt{7}\bar{m}_1)}{k_p[k_a \nu_p \nu_a \nu_\zeta \nu_\epsilon \lambda_{\min}(\boldsymbol{K})\omega_o - \varsigma r_d \sqrt{7}(\rho_1+\rho_2+\rho_3)]} =: r_{ep}^\infty,\end{aligned}$$

where the forms of $\rho_1, \rho_2$, and $\rho_3$ are the same as provided in [1] under Eq. ⟨87⟩.

## References


[1] M.M. Michałek, K. Łakomy, and W. Adamski. Robust output-feedback cascaded tracking controller for spatial motion of anisotropically-actuated vehicles. *Aerosp Sci Technol*, 92:915–929, 2019.