Complementary note on:

Convergence analysis for the orientation error of the unicycle in the VFO set-point control system

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Abstract

This note is a complement to the convergence analysis included in [2] (and also in [1] and [3]). The supplementary material concerns convergence analysis of the unicycle orientation error when the vehicle is controlled in a closed-loop system with the Vector-Field-Orientation (VFO) control law. Formal considerations presented in this note show the terminal convergence of the orientation error to zero in the case of a set-point control task (convergence to the fixed reference orientation).

1 Introduction

The objective of this note is to complement the convergence analysis conducted in [2]. Attention is paid on the convergence of the orientation error e_1 for a case of the set-point control task (considered in Section V of [2] on pages 52-54). In contrast to the original formulation presented in [2] we show convergence of the orientation error e_1 making the analysis by using particular terms expressed in the local frame attached to the reference vehicle (fixed at the reference posture). Notation used in the sequel is almost the same as introduced in [2] in order to preserve coherency of our considerations (only some slight modifications are made for convenience – they will be strictly indicated in Subsection 1.1). The subsequent reasoning will be based on the partial results presented in [2]. References to equations taken from [2] will be denoted by '$\{\cdot\}$'.

Section 3 is devoted to the orientation error convergence analysis in the special case when the finite-time convergence of the position error components together with the control input limitations are assumed. This part of the note complements considerations presented in [1] and [3]. References to equations taken from paper [1] will be denoted by '$< \cdot >$'.

1.1 Notation

The term $\mathrm{sgn}(k)$ introduced in [2], for example in definition {52}, will be now denoted by σ and it will be called the *decision factor* (its value determines the motion strategy (forward/backward) of the controlled vehicle). Note also that the terms $\boldsymbol{e}_p$ and $\boldsymbol{h}_p$ used in this note and in [2] are often denoted in other our publications by $\boldsymbol{e}^*$ and $\boldsymbol{h}^*$, respectively (as in [1] and [3]).

2 Convergence analysis for the orientation error

Let us first redefine the decision factor introduced in {52} as follows:

$$\sigma \triangleq \mathrm{sgn}(e_{20}^t) \equiv \mathrm{sgn}(e_{x0}^t) = \mathrm{sgn}(e_x(0)\cos\varphi_t + e_y(0)\sin\varphi_t) \in \{-1, +1\}, \tag{1}$$

where $e_{20}^t \equiv e_{x0}^t = e_x^t(\tau = 0)$ is an initial error component along x-axis but expressed in the **local frame attached to the reference vehicle** (in contrast to {52} where the initial error component was expressed in the global frame). Definition (1) guarantees convergence of the orientation error for all the vehicle initial conditions defined by Proposition 2 in [2] on page 52.

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In order to show the convergence of the orientation error, recall equation {56} which for $e_a \to 0$ takes the form (we use here the results from {57} and {55}):

$$\dot{\boldsymbol{e}}_p = -k_p \boldsymbol{e}_p - \dot{\boldsymbol{q}}^*_{vt} \tag{2}$$

hence according to {44}, {45} and {47} (with $\rho = \eta$):

$$\dot{e}_x = -k_p e_x + \eta\sigma \, \|\, \boldsymbol{e}_p\| \cos\varphi_t, \tag{3}$$

$$\dot{e}_y = -k_p e_y + \eta\sigma \, \|\, \boldsymbol{e}_p\| \sin\varphi_t. \tag{4}$$

One can express position error $\boldsymbol{e}_p$ in the local frame as follows

$$\boldsymbol{e}^t_p = \begin{bmatrix} e^t_x \\ e^t_y \end{bmatrix} \triangleq \boldsymbol{R}^T \boldsymbol{e}_p = \begin{bmatrix} \cos\varphi_t & \sin\varphi_t \\ -\sin\varphi_t & \cos\varphi_t \end{bmatrix} \begin{bmatrix} e_x \\ e_y \end{bmatrix}, \tag{5}$$

where $\boldsymbol{e}^t_p$ is the vehicle position error expressed in the local frame attached to the reference vehicle. Differentiating (5) with respect to time and using (3)-(4) one obtains (for $e_a = 0$) the dynamics of the position error components expressed in the local frame (see Appendix 4.1):

$$\dot{e}^t_x = -k_p e^t_x + \eta\sigma \, \|\, \boldsymbol{e}_p\| \,, \tag{6}$$

$$\dot{e}^t_y = -k_p e^t_y, \tag{7}$$

where $\|\, \boldsymbol{e}_p\| = \sqrt{e^2_x + e^2_y} \equiv \sqrt{e^{t2}_x + e^{t2}_y}$. According to the above result we can formulate the following corollaries:

C1) $e^t_y(\tau)$ converges to zero independently of $e^t_x(\tau)$ and terminally passes $e^t_x(\tau)$,

C2) using (1) in (6) gives: $\dot{e}^t_x = -k_p e^t_x + \eta \operatorname{sgn}(e^t_{x0}) \,\|\, \boldsymbol{e}_p\|$; hence (since $\eta > 0$) the solution $e^t_x(\tau)$ preserves its initial sign, namely:

$$\forall_{\tau \geq 0} \; \operatorname{sgn}(e^t_x(\tau)) = \operatorname{sgn}(e^t_{x0}), \tag{8}$$

C3) terminally, for $e^t_y \approx 0$, one can write: $\dot{e}^t_x \approx -k_p e^t_x + \eta \operatorname{sgn}(e^t_{x0}) \, |e^t_x|$, and according to (8) one obtains: $\dot{e}^t_x \approx -k_p e^t_x + \eta \operatorname{sgn}(e^t_x) \, |e^t_x| = -k_p e^t_x + \eta \, e^t_x = -(k_p - \eta) e^t_x$. Thus the decay time-constant for solution $e^t_x(\tau)$ is approximately equal to $T_x = 1/(k_p - \eta)$ and for $\eta \in (0, k_p)$ it is lower than $T_y = 1/k_p$ in (7) – terminally, $e^t_x(\tau)$ decays (converges to zero) slower then $e^t_y(\tau)$.

Combining (6)-(7) and C3) allows one to conclude that selection of parameter η simultaneously influences the *passing effect* (the higher η, the earlier $e^t_y(\tau)$ passes $e^t_x(\tau)$ in time) and the resultant convergence rate for $e^t_x(\tau)$ and consequently for $\|\, \boldsymbol{e}_p(\tau)\|$.

The partial results obtained so far will be used later on in the convergence analysis for orientation error e_1. Let us proceed our reasoning by transformation of the convergence vector field $\boldsymbol{h}_p$ into the local reference frame (similarly as it was done for the position error):

$$\boldsymbol{h}^t_p = \begin{bmatrix} h^t_x \\ h^t_y \end{bmatrix} \triangleq \boldsymbol{R}^T \boldsymbol{h}_p = \begin{bmatrix} \cos\varphi_t & \sin\varphi_t \\ -\sin\varphi_t & \cos\varphi_t \end{bmatrix} \begin{bmatrix} h_x \\ h_y \end{bmatrix}, \tag{9}$$

where $\boldsymbol{h}^t$ is the convergence vector field expressed in the local frame attached to the reference vehicle. By defining the angle $\beta = \operatorname{Arg}(\boldsymbol{h}^t_p) \triangleq \operatorname{Atan2}\left(h^t_y, h^t_x\right)$ we obtain:

$$\tan\beta = \frac{h^t_y}{h^t_x} \overset{(9)}{=} \frac{h_y \cos\varphi_t - h_x \sin\varphi_t}{h_y \sin\varphi_t + h_x \cos\varphi_t} = \frac{\left(\frac{h_y}{h_x} - \tan\varphi_t\right) h_x \cos\varphi_t}{\left(\frac{h_y}{h_x} \tan\varphi_t + 1\right) h_x \cos\varphi_t} = \frac{\tan\varphi_a - \tan\varphi_t}{1 + \tan\varphi_a \tan\varphi_t} = \tan(\varphi_a - \varphi_t).$$

Hence, as a direct consequence one can write:

$$\tan(\varphi_a(\tau) - \varphi_t) = \frac{h^t_y(\tau)}{h^t_x(\tau)}. \tag{10}$$

Now, using the above equation we are going to show that $h^t_y(\tau)$ tends to zero faster than $h^t_x(\tau)$ which implies that $\tan(\varphi_a(\tau) - \varphi_t)$ terminally tends toward zero as $\tau \to \infty$. Using (9) and recalling {25} together with {44}, {45}, and {47} (for $\rho = \eta$) one obtains (after simple calculations and using (5)):

$$\boldsymbol{h}^t_p = \begin{bmatrix} h^t_x \\ h^t_y \end{bmatrix} = \begin{bmatrix} \cos\varphi_t & \sin\varphi_t \\ -\sin\varphi_t & \cos\varphi_t \end{bmatrix} \begin{bmatrix} k_p e_x - \eta\sigma \,\|\, \boldsymbol{e}_p\| \cos\varphi_t \\ k_p e_y - \eta\sigma \,\|\, \boldsymbol{e}_p\| \sin\varphi_t \end{bmatrix} = \begin{bmatrix} k_p e^t_x - \eta\sigma \sqrt{e^{t2}_x + e^{t2}_y} \\ k_p e^t_y \end{bmatrix} = \begin{bmatrix} -\dot{e}^t_x \\ -\dot{e}^t_y \end{bmatrix}, \tag{11}$$

where the last equation results from (6) and (7). Hence, we can rewrite (10) as

$$\tan(\varphi_a(\tau) - \varphi_t) = \frac{h_y^t(e_y^t(\tau))}{h_x^t(e_x^t(\tau), e_y^t(\tau))} = \frac{-\dot{e}_y^t(\tau)}{-\dot{e}_x^t(\tau)} = \frac{k_p e_y^t(\tau)}{k_p e_x^t(\tau) - \eta\sigma\sqrt{e_x^{t2}(\tau) + e_y^{t2}(\tau)}}. \tag{12}$$

Now, since $e_x^t(\tau) \to 0$ and $e_y^t(\tau) \to 0$ for $\tau \to \infty$[1] but, according to corollary C1), $e_y^t(\tau)$ terminally passes $e_x^t(\tau)$ one concludes that

$$\tan(\varphi_a(\tau) - \varphi_t) \to 0 \quad \text{as} \quad \tau \to \infty. \tag{13}$$

Relation (13) together with the convergence result $(\varphi_a(\tau) - \varphi(\tau)) \to 0$ (see {55}) allows concluding that

$$\tan(\varphi(\tau) - \varphi_t) \to 0 \quad \text{as} \quad \tau \to \infty. \tag{14}$$

Obviously, (14) still does not mean that $e_1(\tau) = f_e(\varphi_t(\tau) - \varphi) \to 0$ for $\tau \to \infty$, since (14) can be met also for $e_1(\tau) \to \pm\pi$. However, it can be additionally shown that terminally for $e_a(\tau), e_y(\tau), e_x(\tau) \to 0$ (or equivalently for $e_a(\tau), e_y^t(\tau), e_x^t(\tau) \to 0$) the sign $\text{sgn}(\boldsymbol{g}_{2t}^{*T}\boldsymbol{g}_2^*) = \text{sgn}(\cos(\varphi - \varphi_t)) = \text{sgn}(\cos(\varphi_t - \varphi))$ is equal to $+1$, which along with (14) will allow concluding terminal convergence $(\varphi(\tau) - \varphi_t) \to 0 \bmod 2\pi$, and consequently convergence of $e_1(\tau)$ toward zero. Since $\boldsymbol{g}_2^* = \dot{\boldsymbol{q}}^*/u_2 = [\dot{x}/u_2 \ \dot{y}/u_2]^T$ (see {11}) one can write:

$$\text{sgn}(\boldsymbol{g}_{2t}^{*T}\boldsymbol{g}_2^*) = \text{sgn}(u_2)\text{sgn}(\dot{x}\cos\varphi_t + \dot{y}\sin\varphi_t).$$

Using the fact that $u_2 = \sigma \| \boldsymbol{h}_p \| \cos e_a$ (see Appendix 4.2) we write the above equation at the limit for $e_a \to 0$ as[2]:

$$\begin{aligned}\lim_{e_a \to 0} \text{sgn}(\boldsymbol{g}_{2t}^{*T}\boldsymbol{g}_2^*(\varphi_a - e_a)) &= \lim_{e_a \to 0} \text{sgn}(u_2(e_a))\text{sgn}(\dot{x}(e_a)\cos\varphi_t + \dot{y}(e_a)\sin\varphi_t) = \\ &= \sigma\,\text{sgn}(h_x \cos\varphi_t + h_y \sin\varphi_t) \overset{(9)}{=} \sigma\,\text{sgn}(h_x^t) = \\ &\overset{(11)}{=} \sigma\,\text{sgn}(k_p e_x^t - \eta\sigma\sqrt{e_x^{t2} + e_y^{t2}}),\end{aligned} \tag{15}$$

where $\boldsymbol{g}_2^*(\varphi) = \boldsymbol{g}_2^*(\varphi_a - e_a)$ since $e_a \triangleq \varphi_a - \varphi$. Because $e_y^t(\tau)$ terminally passes $e_x^t(\tau)$ we can write

$$\begin{aligned}\lim_{e_x^t, e_y^t \to 0}\left(\lim_{e_a \to 0} \text{sgn}(\boldsymbol{g}_{2t}^{*T}\boldsymbol{g}_2^*)\right) &\overset{(15)}{=} \lim_{e_x^t \to 0}\lim_{e_y^t \to 0} \sigma\,\text{sgn}(k_p e_x^t - \eta\sigma\sqrt{e_x^{t2} + e_y^{t2}}) = \\ &= \lim_{e_x^t \to 0} \sigma\,\text{sgn}(k_p e_x^t - \eta\sigma\sqrt{e_x^{t2}}) = \lim_{e_x^t \to 0} \sigma\,\text{sgn}(k_p e_x^t - \eta\sigma\left|e_x^t\right|) = \\ &\overset{(1)}{=} \lim_{e_x^t \to 0} \text{sgn}(e_{x0}^t)\,\text{sgn}(k_p e_x^t - \eta\,\text{sgn}(e_{x0}^t)\left|e_x^t\right|) = \\ &\overset{(8)}{=} \lim_{e_x^t \to 0} \text{sgn}(e_{x0}^t)\,\text{sgn}(k_p e_x^t - \eta\,\text{sgn}(e_x^t)\left|e_x^t\right|) = \\ &= \lim_{e_x^t \to 0} \text{sgn}(e_{x0}^t)\,\text{sgn}(e_x^t)\,\text{sgn}(k_p - \eta) = \\ &\overset{(8)}{=} \lim_{e_x^t \to 0} \text{sgn}^2(e_{x0}^t)\,\text{sgn}(k_p - \eta) = 1,\end{aligned} \tag{16}$$

where in the last stage we have used the fact that $\eta \in (0, k_p)$ from assumption (see {46}).

Combination of the results (14) and (16) allows concluding about terminal convergence of the orientation error $e_1(\tau) \triangleq f_e(\varphi_t - \varphi(\tau)) \in \mathbb{S}^1$ (see {8}):

$$e_1(\tau) \to 0 \quad \text{as} \quad \tau \to \infty. \tag{17}$$

It is consistent with the statement presented in our original work [2] on page 54.

Remark 1 *In (1) we have introduced an alternative definition for the decision variable σ (in comparison to eq. {52} in [2]), which allowed us to show the terminal convergence of the orientation error e_1 when $\| \boldsymbol{e}_p \| \to 0$ ($\epsilon := 0$ in {62}). However, it is not always required to use (1) for motion strategy selection. In fact, the decision factor σ can be freely selected regardless of the vehicle initial condition excluding the special case, which is related to situation where*

$$e_y^t(0) = 0 \quad \wedge \quad e_1(0) = \pm\pi.$$

In this case usage of definition (1) ensures terminal convergence of the orientation error as stated in (17), while usage of definition proposed in {52} generally does not.

[1] It is evident using (5) and since $\| \boldsymbol{e}_p(\tau) \| \to 0$ as $\tau \to \infty$ (see {59}).

[2] According to {31} $e_a \to 0$ arbitrarily fast and independently of $\boldsymbol{e}^*(\tau)$.

3 Convergence of the orientation error under conditions of the finite-time position error convergence and in the presence of control input limitations

In this section we use the original notation introduced in [1], namely: $\boldsymbol{e}^*$ for the position error vector, and $\boldsymbol{h}^*$ for the convergence vector field. However, the term sgnU_2 used in [1] is replaced here by the symbol σ and it is called the *decision factor*. According to equation $< 27 >$ and definition $< 15 >$ formulated in [1] we obtain the following position error dynamics valid for $e_a \to 0$:

$$\dot{e}_x = \rho\, s\,(-k_p e_x + \eta\sigma \,\|\,\boldsymbol{e}^*\| \cos\varphi_t), \tag{18}$$
$$\dot{e}_y = \rho\, s\,(-k_p e_y + \eta\sigma \,\|\,\boldsymbol{e}^*\| \sin\varphi_t), \tag{19}$$

where σ is now defined in (1), $\rho = \rho(\|\,\boldsymbol{e}^*\|)$ and $s \in (0, 1]$ are some scalar strictly positive functions. Function ρ is responsible for the finite-time convergence of e_x and e_y to zero, while s is a scaling function resulting from the presence of control input limitations (see [1] and [3] for details). Clearly, (18)-(19) are similar to (3)-(4), thus using the transformation (5) yields:

$$\dot{e}^t_x = \rho\, s\,(-k_p e^t_x + \eta\sigma \,\|\,\boldsymbol{e}^*\|), \tag{20}$$
$$\dot{e}^t_y = \rho\, s\,(-k_p e^t_y). \tag{21}$$

Since s and ρ are strictly positive, the corollary C1) and condition (8) remain valid also for dynamics (20)-(21). In the case of finite-time convergence the corollary C1) leads to:

$$\lim_{\tau\to\tau_x} e^t_x(\tau) = 0, \qquad \lim_{\tau\to\tau_y} e^t_y(\tau) = 0, \qquad \tau_y < \tau_x < \infty, \tag{22}$$

where τ_x and τ_y are the finite time-instants. Using (11), one can obtain the same relation as in (12), namely:

$$\tan(\varphi_a(\tau) - \varphi_t) = \frac{k_p e^t_y(\tau)}{k_p e^t_x(\tau) - \eta\sigma\sqrt{e^{t2}_x(\tau) + e^{t2}_y(\tau)}}. \tag{23}$$

Due to the finite-time convergence (22) and due to the form of (23) relation (13) takes in the considered case the following form:

$$\tan(\varphi_a(\tau) - \varphi_t) \to 0 \quad \text{as} \quad \tau \to \tau_y, \qquad \tau_y < \infty. \tag{24}$$

Since the error $e_a(\tau) = (\varphi_a(\tau) - \varphi(\tau)) \to 0$ in finite time as $\tau \to \tau_a$ (see $< 26 >$ in [1]) we can rewrite (24) as

$$\tan(\varphi(\tau) - \varphi_t) \to 0 \quad \text{as} \quad \tau \to \tau^*, \qquad \tau^* \le \tau_y + \tau_a. \tag{25}$$

Since C1) and (8) remain valid in the finite-time case, relation (16) holds[3] also in case of dynamics (20)-(21):

$$\lim_{e_a\to 0} \mathrm{sgn}(\boldsymbol{g}^{*T}_{2t}\, \boldsymbol{g}^*_2(\varphi_a - e_a)) = \sigma\, \mathrm{sgn}(k_p e^t_x - \eta\sigma\sqrt{e^{t2}_x + e^{t2}_y}) \tag{26}$$

and

$$\lim_{e^t_x, \to 0}\lim_{e^t_y\to 0}\left(\lim_{e_a\to 0} \mathrm{sgn}(\boldsymbol{g}^{*T}_{2t}\, \boldsymbol{g}^*_2(\varphi_a - e_a))\right) = \lim_{e^t_x\to 0} \mathrm{sgn}^2(e^t_{x0})\, \mathrm{sgn}(k_p - \eta) = 1, \tag{27}$$

where the intermediate computations (omitted here) are analogous as for the infinite-time case. Note that now relation (26) holds in the finite time for $\tau \ge \tau_a$, where $\tau_a < \infty$ has been estimated in $< 26 >$.

According to the above reasoning the conclusion (17) is preserved, but now within the finite-time horizon $[0, \tau^*]$, namely:

$$e_1(\tau) \to 0 \quad \text{as} \quad \tau \to \tau^*, \quad \text{where} \quad \tau^* \le \tau_y + \tau_a < \infty \tag{28}$$

with τ_y introduced in (22) and τ_a estimated in $< 26 >$.

Obviously, in the special case for $s \equiv 1$ (lack of control input limitations) all the analysis conducted so far along with the main conclusion (28) remain valid.

[3] Note that now $u_2 = s\,\rho\,\|\,\boldsymbol{h}^*\| \cos\alpha = \sigma\, s\,\rho\,\|\,\boldsymbol{h}^*\| \cos e_a$ and s, ρ are strictly positive functions (see $< 22 >$), thus $\mathrm{sgn}(u_2) = \mathrm{sgn}(\sigma)$ for $e_a \to 0$.

4 Appendix

4.1 Derivation of relations (6)-(7)

Differentiating with respect to time the particular rows of (5) and using the right-hand sides of (3)-(4) one obtains:

$$\begin{aligned}\dot{e}_x^t &= \dot{e}_x \cos\varphi_t + \dot{e}_y \sin\varphi_t = \\ &= (-k_p e_x + \eta\sigma \,\|\boldsymbol{e}_p\| \cos\varphi_t)\cos\varphi_t + (-k_p e_y + \eta\sigma\,\|\boldsymbol{e}_p\|\sin\varphi_t)\sin\varphi_t = \\ &= -k_p(e_x\cos\varphi_t + e_y\sin\varphi_t) + \eta\sigma\,\|\boldsymbol{e}_p\| = -k_p e_x^t + \eta\sigma\,\|\boldsymbol{e}_p\|\end{aligned}$$

and

$$\begin{aligned}\dot{e}_y^t &= -\dot{e}_x \sin\varphi_t + \dot{e}_y \cos\varphi_t = \\ &= -(-k_p e_x + \eta\sigma\,\|\boldsymbol{e}_p\|\cos\varphi_t)\sin\varphi_t + (-k_p e_y + \eta\sigma\,\|\boldsymbol{e}_p\|\sin\varphi_t)\cos\varphi_t = \\ &= -k_p(-e_x\sin\varphi_t + e_y\cos\varphi_t) = -k_p e_y^t.\end{aligned}$$

4.2 Alternative expression for control signal u_2

Let us start with an alternative form of u_2 and try to obtain, after simple manipulations and using $e_a \triangleq \varphi_a - \varphi$, its original form proposed in [2]:

$$\begin{aligned}u_2 &:= \sigma\,\|\boldsymbol{h}_p\|\cos e_a = \sigma\,\|\boldsymbol{h}_p\|\cos(\varphi_a - \varphi) = \sigma\,\|\boldsymbol{h}_p\|\,(\cos\varphi_a\cos\varphi + \sin\varphi_a\sin\varphi) = \\ &= \sigma\,\|\boldsymbol{h}_p\|\,[\cos\varphi_a \;\; \sin\varphi_a]\begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix} = \sigma\,\|\boldsymbol{h}_p\|\begin{bmatrix}\dfrac{\sigma\, h_x}{\|\boldsymbol{h}_p\|} & \dfrac{\sigma\, h_y}{\|\boldsymbol{h}_p\|}\end{bmatrix}\boldsymbol{g}_2^* = \sigma^2\,\|\boldsymbol{h}_p\|\,\frac{\boldsymbol{h}_p^T\boldsymbol{g}_2^*}{\|\boldsymbol{h}_p\|} = \\ &= \|\boldsymbol{h}_p\|\,\frac{\boldsymbol{h}_p^T\boldsymbol{g}_2^*}{\|\boldsymbol{h}_p\|\,\|\boldsymbol{g}_2^*\|} = \|\boldsymbol{h}_p\|\,\frac{\boldsymbol{g}_2^{*T}\boldsymbol{h}_p}{\|\boldsymbol{g}_2^*\|\,\|\boldsymbol{h}_p\|} = \|\boldsymbol{h}_p\|\cos\alpha,\end{aligned}$$

where $\alpha = \angle(\boldsymbol{g}^*, \boldsymbol{h}_p)$. Recalling definition {22} and formula {23} presented in [2] we can see that the two considered alternative forms of the control input u_2 are equivalent.

References

[1] M. Michałek and K. Kozłowski. Finite-time VFO stabilizers for the unicycle with constrained control input. In *Robot Motion and Control 2009*, volume 396 of *Lecture Notes in Control and Information Sciences*, pages 23–34. Springer, 2009.

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